Accuracy, Consistency, and Stability of Measurements of h Enabling the Redefinition of the Kilogram

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World Metrology Day 2018
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NIST

Kilogram (U.S. National Prototype) — Current SI

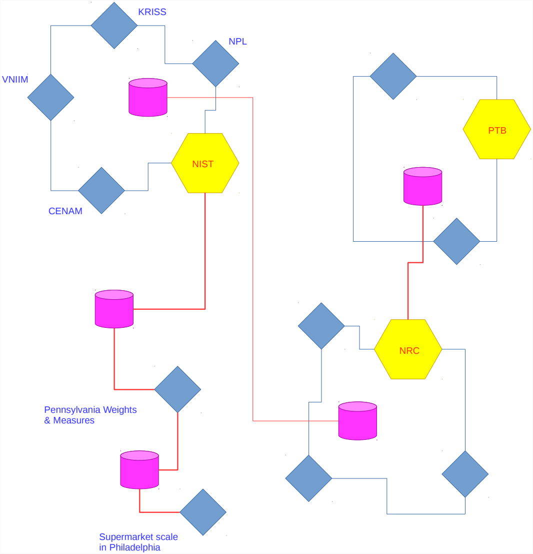
K20

Kilogram (Revised SI)

$$1\,\mathrm{kg} = \frac{(299\,792\,458)^2}{(9\,192\,631\,770)(6.626\,070\,15 \times 10^{-34})} \frac{\Delta\nu_{\mathrm{Cs}} h}{c^2}$$

$\Delta\nu_{\mathrm{Cs}}$	Frequency of hyperfine transition in unperturbed ground state of ^{133}Cs	(1967)
c	Speed of light in vacuum	(1983)
h	Planck constant	(2018)

Mass Measurement (Revised SI)



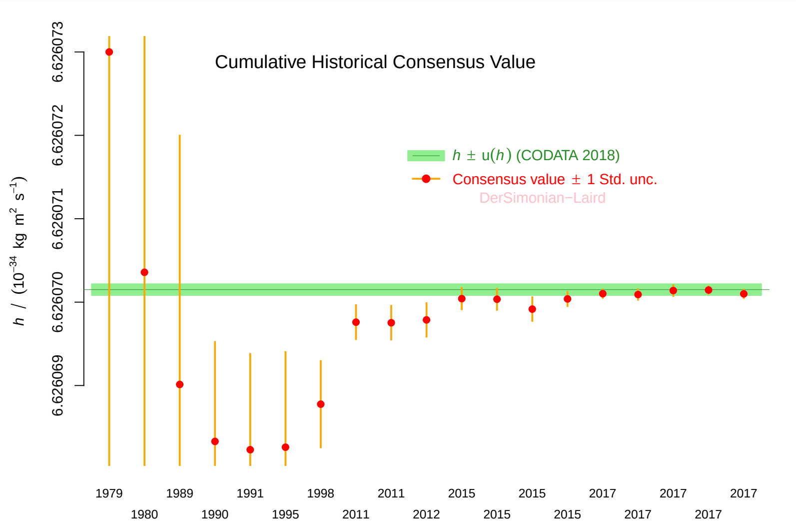
CCM Recommendation G1 — May 2017

CIPM undertakes the necessary steps to proceed with the planned redefinition of the SI at the next meeting of the CGPM, acknowledging the measures to be taken by the CCM to ensure integrity and continuity in the dissemination of the kilogram

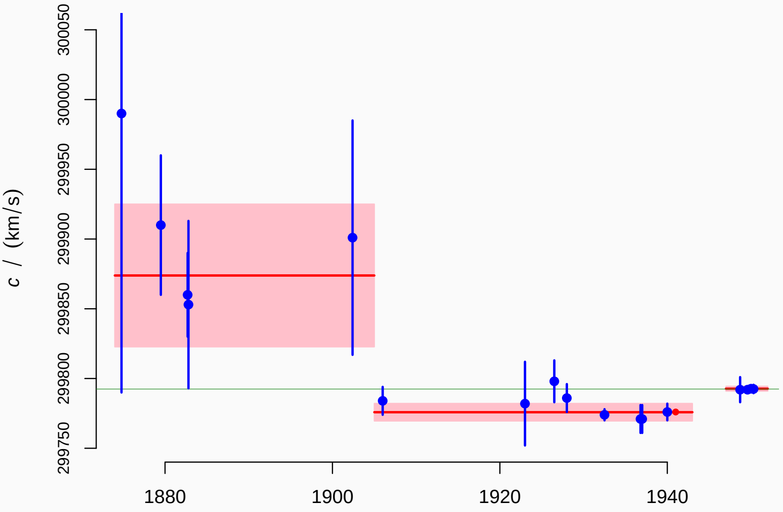
CCU Recommendation U1 — September 2017

CIPM undertakes the necessary steps to proceed with the planned redefinition of the kilogram, ampere, kelvin and mole at the 26th CGPM in 2018

Planck Constant — 40 Year Retrospective



Speed of Light — 75 Year Retrospective



CCM Conditions for Redefinition (1/3)

- At least three independent experiments, including work from Kibble balance and XRCD experiments, yield values of the Planck constant with relative standard uncertainties not larger than 5×10^{-8}
- At least one of these results should have a relative standard uncertainty not larger than 2×10^{-8}

CCM Conditions for Redefinition (2/3)

- The BIPM prototypes, the BIPM ensemble of reference mass standards, and the mass standards used in the Kibble balance and XRCD experiments have been compared as directly as possible with the international prototype of the kilogram
  - Pilot Study CCM.R-kg-P1: weighted mean of measurement results agreed with calibration based on IPK to within 0.001 mg for one set of 1 kg standards, and to within 0.0045 mg for the other

CCM Conditions for Redefinition (3/3)

- The procedures for the future realization and dissemination of the kilogram, as described in the *mise en pratique*, have been validated in accordance with the principles of the CIPM MRA

Planck Constant — Measurement Results

	$h/10^{-34}\,\mathrm{J\,s}$	$u(h)/10^{-41}\,\mathrm{J\,s}$	$(u(h)/h)/10^{-8}$	
NPL-79	6.626073	67	101	E
IAC-11	6.62606989	2	3	X
METAS-11	6.6260691	20	30	K
NPL-12	6.6260712	13	20	K
IAC-15	6.62607015	1.3	2	X
LNE-15	6.6260688	17	26	K
NIST-15	6.62606936	3.8	6	K
NRC-17	6.626070133	0.6	1	K
LNE-17	6.62607041	3.8	6	K
NMIJ-17	6.626070059	1.65	2	X
NIM-17	6.6260692	16	24	J
IAC-17	6.626070404	0.792	1	X
NIST-17	6.626069934	0.89	1	K

Planck Constant — Inconsistency

> *most recent measurement results with relative standard uncertainty below 5×10^{-8} do not pass the standard chi-squared test of consistency*
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  - When many measurement results are involved, test may declare statistically significant heterogeneity that is substantively irrelevant
  - Validity hinges on assumption that uncertainties are all based on infinitely many degrees of freedom

Impact of Potential Inconsistency (1/2)

	$h/10^{-34}\,\mathrm{J\,s}$	$(u(h)/h)/10^{-8}$	
IAC-11	6.62606989	3	X
IAC-15	6.62607015	2	X
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- Leave one out in turn and compute consensus value of the other five: relative standard deviation of resulting six consensus values is 6×10^{-9}
- Maximum relative permissible error for class E_1 standard of mass 1 kg is 83×10^{-9}

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- Following *Extraordinary Calibrations* of 2013-2014, BIPM “as-maintained mass unit” shifted by 35 µg
  - Shift was average mass loss of BIPM working standards relative to IPK, between 1992 and 2014

Reassessing Inconsistency — Birge's Model

$$\boxed{h_j = h + \kappa\varepsilon_j}$$

- h — **True value** of the Planck constant
- h_j — Value of h **measured** in experiment j
- ε_j — **Measurement error** in experiment j, Gaussian with mean 0 and standard deviation u_j
- κ — **Adjustment factor** that "corrects" reported uncertainties when they appear to be too low

Reassessing Inconsistency — Birge's Adjustment

$h_j = h + \kappa\varepsilon_j$

- κ often estimated by *Birge Ratio* (1.9 in this case) without recognizing associated uncertainty
- Maximum likelihood estimate
  - $\widehat{\kappa} = 1.745$
  - 95 % coverage interval $(0.65, 2.50)$, computed using parametric statistical bootstrap, includes 1

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➠ No reason to reject hypothesis of homogeneity

Homogeneous Triplets

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- Set of six experiments with rel. std. unc. $\leqslant 5 \times 10^{-8}$ has 20 subsets of size 3
- Nine subsets are homogeneous (Cochran's chi-squared test), and comprise:
  - One experiment with rel. std. unc. $\leqslant 2 \times 10^{-8}$
  - One Kibble balance experiment
  - One XRCD experiment

CODATA Recommended Values

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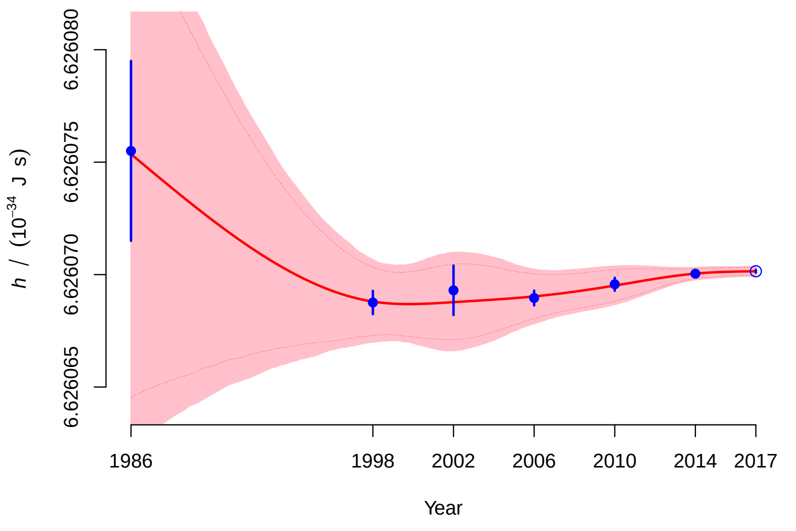
  - This was not among the preconditions that the CCM deemed necessary for redefinition

CODATA Recommended Values

- Gläser, Borys, Ratschko & Schwartz (2010) argued that
  - $u(h)/h \leqslant 30 \times 10^{-9}$ would be acceptable
  - If the CCM requirements are met, no serious changes in the calibration chain of mass standards would occur
  - No mention of any requirement involving sequence of CODATA recommended values

30 Years of CODATA Recommendations (1/2)



Correlations — Evaluation & Propagation

2006					2010				
h_{NPL79}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NPL79}}$	h_{NPL79}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NPL79}}$
h_{NIST80}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NIST80}}$	h_{NIST80}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NIST80}}$
h_{NMI89}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NMI89}}$	h_{NMI89}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NMI89}}$
h_{NPL90}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NPL90}}$	h_{NPL90}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NPL90}}$
h_{PTB91}	$=$	h_{2006}	$+$	$\varepsilon_{\text{PTB91}}$	h_{PTB91}	$=$	h_{2010}	$+$	$\varepsilon_{\text{PTB91}}$
h_{NIM95}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NIM95}}$	h_{NIM95}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NIM95}}$
h_{NIST98}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NIST98}}$	h_{NIST98}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NIST98}}$
h_{NPI05}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NPI05}}$					
h_{NIST07}	$=$	h_{2006}	$+$	$\varepsilon_{\text{NIST07}}$	h_{NIST07}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NIST07}}$
					h_{IAC11}	$=$	h_{2010}	$+$	$\varepsilon_{\text{IAC11}}$
					h_{METAS11}	$=$	h_{2010}	$+$	$\varepsilon_{\text{METAS11}}$
					h_{NPL12}	$=$	h_{2010}	$+$	$\varepsilon_{\text{NPL12}}$

- Correlation between $\widehat{h}_{2006}$ and $\widehat{h}_{2010}$: $r_{2006,2010} = 0.37$
- Number of degrees of freedom (8) recognized via asymptotic distribution of Fisher's z transform
- Correlations propagated by Monte Carlo sampling from multivariate Gaussian distribution

30 Years of CODATA Recommendations (2/2)

- Uncertainty band takes correlations into account and fully accommodates horizontal line across whole range of dates on horizontal axis
  - ➠ Insufficient evidence to reject hypothesis of no trend

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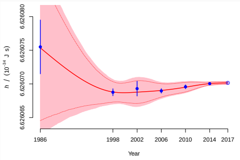
  - ➠ Insufficient evidence to reject hypothesis of no trend

- Relative value of slope (first derivative) of red trend line at 2017 is 2 ± 6 parts in 10^9 per year
  - ➠ Slope at 2017 does not differ significantly from 0



CODATA 2017 — Newell *et al.* (2018)

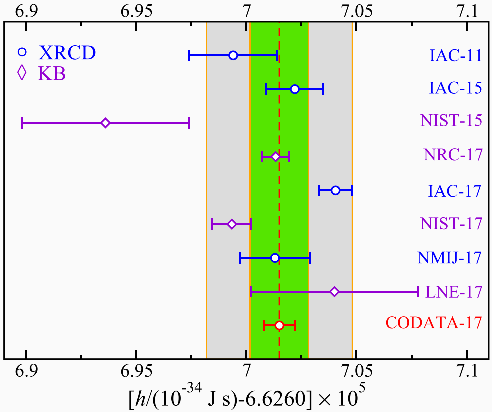


Figure 1. Values of the Planck constant h inferred from the input data in table 1 and the CODATA 2017 value in chronological order from top to bottom. The inner green band is ± 20 parts in 10^9 and the outer grey band is ± 50 parts in 10^9. KB: Kibble balance; XRCD: x-ray-crystal-density.

Residual Uncertainty

- CODATA 2017 Special Adjustment

$$h = 6.626\,070\,150(69) \times 10^{-34}\ \mathrm{J\,s}$$

- Relative standard uncertainty 1.0×10^{-8}

- Upon redefinition, mass of IPK will be 1 kg to within this relative standard uncertainty

  Thereafter its value will be determined experimentally

  — Draft Resolution A, 26th meeting, CGPM, November 2018

BIPM Calibration — 1 kg Mass Prototype No. 85

- Certificate dated August 1st, 2012
- Certified mass 1 kg − 0.743 mg
- Combined standard uncertainty 0.007 mg
  - Relative standard uncertainty 0.7×10^{-8}
    - Compare with rel. std. unc. 1.0×10^{-8} associated with CODATA 2017 recommended value for h

Agreement

- Agreement between most accurate determinations of h available today is remarkable
  - Especially considering that they involve multiple, independent implementations of two completely different measurement procedures

Absolute Realization

- Both Kibble balance and XRCD method provide absolute realization of the unit of mass
  - For the first time since 1795, when National Convention of First Republic of France passed decree defining the gram as

    > *absolute weight of a volume of pure water equal to the cube of the one hundredth part of a meter, at the temperature of melting ice*

Redefinition — Accomplishments

- Accomplishments in measurement of h are far reaching
  - Create opportunity for many laboratories, not just NMIs — in national and local government agencies, universities, and industry — to realize the unit of mass

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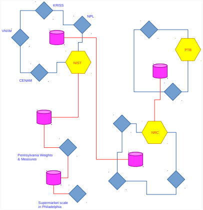
  - Create opportunity for many laboratories, not just NMIs — in national and local government agencies, universities, and industry — to realize the unit of mass
  - Facilitate and reduce time and costs to disseminate unit throughout network of traceability chains anchored on multiple primary realizations



Redefinition — Collective Effort

- No participant has been left out or behind in this vast enterprise
  - All have added tangible value, regardless of how any particular measurement result may be situated relative to the consensus value that blends them all and enables reliable redefinition of the kilogram